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Analysis of an infinite shape memory alloy plate with a circular hole subjected to biaxial tension†

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Abstract

This paper presents an exact solution for the stresses in an infinite shape memory alloy plate with a circular hole subjected to biaxial tensile stresses applied at infinity. The solution obtained by assumption of plane stress is based on the two-dimensional version of the Tanaka constitutive law for shape memory materials. The plate is in the austenitic phase, prior to the application of external stresses. However, as a result of tensile loading, stress-induced martensite forms, beginning from the boundary of the hole and extending into the interior, as the load continues to increase. Therefore, in a general case, the plate consists of three annular regions: the inner region of pure martensite, the intermediate region where martensite and austenite coexist, and the outer region of pure austenite. The boundaries between these annular regions can be found as functions of the external stress. Two methods of solution are presented. The first is a closed-form approach based on a replacement of the actual distribution of the martensitic fraction by a piece-wise constant function of the radial coordinate. The second method results in an exact solution obtained by assuming that the ratio between the radial and circumferential stresses in the region where austenite and martensite coexist is governed by the same relationship as that in the encompassing regions of pure austenite and pure martensite.

1. Introduction

Problems of a stress distribution in plates with discontinuities represent a significant interest both for fundamental mechanics and in practical engineering. One of the classical problems that was considered in the sixties and seventies is an inelastic stress concentration in an infinite plate with a circular hole subjected to an equal biaxial tension. Mentioned here are the papers of Budiansky and Mangasarian (1960) and Budiansky (1971) who employed the J_2 deformation theory to obtain an exact solution for a stress concentration factor in elasto-plastic plates. In the

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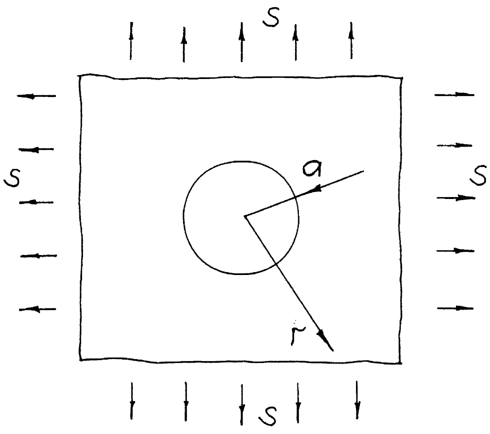

Fig. 1. The plate subjected to biaxial tension.

present paper, the problem of a distribution of stresses in plates with a circular hole is extended to the plates from shape memory materials subjected to an isothermal loading. The process of martensitic transformation considered in the paper is characterized by a nonlinear stress–strain relationship, i.e. the classical elasticity solution becomes invalid. However, the paper illustrates a closed-form solution that can be obtained by subdividing the plate into annular regions where the martensitic fraction and effective stress remain constant. In addition, an exact solution is found for the entire plate, including the region where martensite and austenite coexist. This solution is obtained by assuming that the relationship between the radial and circumferential stresses in the region of mixed martensite and austenite is governed by the same law as in the regions of pure martensite and pure austenite.

2. Analysis

Consider an infinite plate from a shape memory material with a circular hole subjected to an equal biaxial tension, as shown in Fig. 1. The plate is assumed in the state of plane stress, i.e. only the radial (σ_r) and circumferential (σ_θ) components of the stress tensor are present. In the absence of an external load, the material is in the austenitic phase. Then the load is gradually increased, while the temperature is kept constant. Eventually, as the effective stress along the boundary of the hole reaches a critical value, the stress-induced martensitic transformation is triggered. At a certain value of the effective stress the material around the hole is entirely converted into martensite and the plate consists of three regions: the inner martensitic region limited by a $< r < r_m$, the intermediate region of martensite mixed with austenite within $r_m < r < r_a$ and the outer austenitic region at $r > r_a$.

2.1. *Two-dimensional constitutive equations for shape memory alloys*

The solution is based on the constitutive theory of Tanaka (1990) that was generalized by Boyd and Lagoudas (1993) for a three-dimensional case. According to this theory, the constitutive equations of a shape memory material can be written as

$$\sigma = C(\varepsilon - \alpha\Delta T - \varepsilon^{t}) \tag{1}$$

where σ, ε and ε^{t} are vectors of stresses, elastic strains and transformation strains, respectively, C is a matrix of instantaneous elastic stiffnesses, α is a vector of instantaneous coefficients of thermal expansion and ΔT is a variation of temperature from the stress-free reference value. The instantaneous stiffnesses and coefficients of thermal expansion depend on the state variables, i.e. the strains, temperature and the martensitic fraction ξ.

In the case where a phase transformation occurs without a reorientation of martensitic variants, it is assumed that the rate of the transformation strain is proportional to the rate of change of the martensitic fraction (Bondaryev and Wayman, 1988; Boyd and Lagoudas, 1997), i.e.

$$d\varepsilon^{t} = \Lambda d\xi \tag{2}$$

where Λ is a transformation tensor related to the deviatoric strain components. Following the paper of Birman et al. (1966), the transformation tensor components are taken proportional to the maximum transformation strain observed in a one-dimensional test. Then eqn (2) can be integrated and the axisymmetric plane stress constitutive relations for an isothermal loading become:

$$\begin{aligned} \sigma_{r} &= C_{rr}\varepsilon_{r} + C_{r\theta}\varepsilon_{\theta} - \Lambda_{r}(C_{rr} + C_{r\theta})\xi \\ \sigma_{\theta} &= C_{\theta r}\varepsilon_{r} + C_{\theta\theta}\varepsilon_{\theta} - \Lambda_{\theta}(C_{\theta r} + C_{rr})\xi \end{aligned} \tag{3}$$

where, if the material is isotropic, $C_{rr} = C_{\theta\theta} = D$, $C_{r\theta} = C_{\theta r} = vD$, $D = D(\varepsilon, T, \xi)$ is an instantaneous elastic modulus, and v is the Poisson ratio which is assumed to be constant. In isotropic materials, $\Lambda_{r} = \Lambda_{\theta} = \omega$ and the constitutive law can be written as

$$\begin{aligned} \sigma_{r} &= D[\varepsilon_{r} + v\varepsilon_{\theta} - \omega(1+v)\xi] \\ \sigma_{\theta} &= D[v\varepsilon_{r} + \varepsilon_{\theta} - \omega(1+v)\xi] \end{aligned} \tag{4}$$

The inverse of eqn (4) is

$$\begin{aligned} \varepsilon_{r} &= (\sigma_{r} - v\sigma_{\theta})/[(1-v^{2})D] + \omega\xi \\ \varepsilon_{\theta} &= (\sigma_{\theta} - v\sigma_{r})/[(1-v^{2})D] + \omega\xi \end{aligned} \tag{5}$$

These constitutive equations must be considered together with the transformation kinetics and the nucleation criterion. According to Tanaka's constitutive theory, the transformation kinetics equation corresponding to the martensitic transformation is:

$$\xi = 1 - \exp\,[b_{M}(M_{S}{}^{\circ} - T) + (b_{M}/d_{M})\sigma] \tag{6}$$

where σ is an effective stress, $M_{S}{}^{\circ}$ is the martensite start temperature corresponding to the stress-free state, T is a current temperature, and d_{M} is a slope of the martensite transformation temperature lines in the stress-temperature plane. Note that in the present paper this slope is assumed identical in the planes "effective stress—temperature" and "one-dimensional stress—temperature". The constant b_{M} is a function of the martensite start and finish ($M_{F}{}^{\circ}$) temperatures in the absence of stresses:

$$b_{M} = \ln 0.01/(M_{S}{}^{\circ} - M_{F}{}^{\circ}) \tag{7}$$

The effective stress is chosen as in three-dimensional SMA constitutive theories of Liang and Rogers (1991) and Boyd and Lagoudas (1993):

$$\sigma = [(3/2)\sigma'_{ij}\sigma'_{ij}]^{1/2} \tag{8}$$

where σ'_{ij} is a component of the deviatoric stress tensor. In the present problem, the effective stress given by (8) becomes

$$\sigma = (\sigma_r^2 + \sigma_\theta^2 - \sigma_r\sigma_\theta)^{1/2} \tag{9}$$

The conditions corresponding to the start of the martensitic transformation can be obtained from the transformation kinetics eqn (6) with $\xi = 0$. The transformation is assumed accomplished when $\xi = 0.99$. Accordingly, the range of the effective stresses corresponding to the martensitic transformation coincides with that for a one-dimensional problem, i.e.

$$d_M(T - M_S^\circ) < \sigma < d_M \ln 0.01/b_M + d_M(T - M_S^\circ) \tag{10}$$

2.2. *Closed-form solution of the problem*

When the effective stress is below the value given by the left side of inequality (10), the elastic solution yields (Budiansky, 1971)

$$\sigma_r = s[1 - (a/r)^2]$$

$$\sigma_\theta = s[1 + (a/r)^2] \tag{11}$$

where r is a radial coordinate, a is a radius of the hole and s is the stress applied at infinity where $\sigma_r = \sigma_\theta = s$. The martensitic transformation starts at the value of the external stress that can be found from eqns (9) and (11) and the left side of inequality (10):

$$s = d_M(T - M_S^\circ)/\{[1 + 3(a/r)^4]^{1/2}\} \tag{12}$$

Accordingly, the transformation begins at the boundary of the hole when $s = d_M(T - M_S^\circ)/2$. The transformation will spread over the entire plate when $s = d_M(T - M_S^\circ)$.

Once the transformation has started, additional external stresses will result in an expansion of the region where the material is partially transformed into martensite. At each value of s, the boundary between the region containing a mixture of austenite and martensite and the outer austenitic region, i.e. r_a, is given by (12).

The inner boundary of the austenitic region is shown in Fig. 2 for three different values of temperature. The properties of the material are (Lei and Wu, 1991):
$D_A = 30$ GPa (modulus of elasticity of austenite); $D_M = 13$ GPa (modulus of elasticity of martensite); $v = 0.33$; $M_S^\circ = 23^\circ$C; $M_F^\circ = 5^\circ$C; $d_M = 11.3$ MPa/°C; $\omega = 0.07$.

The nondimensional variables in Fig. 2 are:

$$S = 10^3\, s/D_A, \quad R_a = r_a/a \tag{13}$$

As follows from Fig. 2, for a prescribed value of the external stress, the extent of the region where the material has partially or completely converted into martensite decreases as the temperature increases. Of course, this is a logical conclusion since it is well known that stress-induced martensite can not be obtained at high temperatures (Wayman and Duerig, 1990).

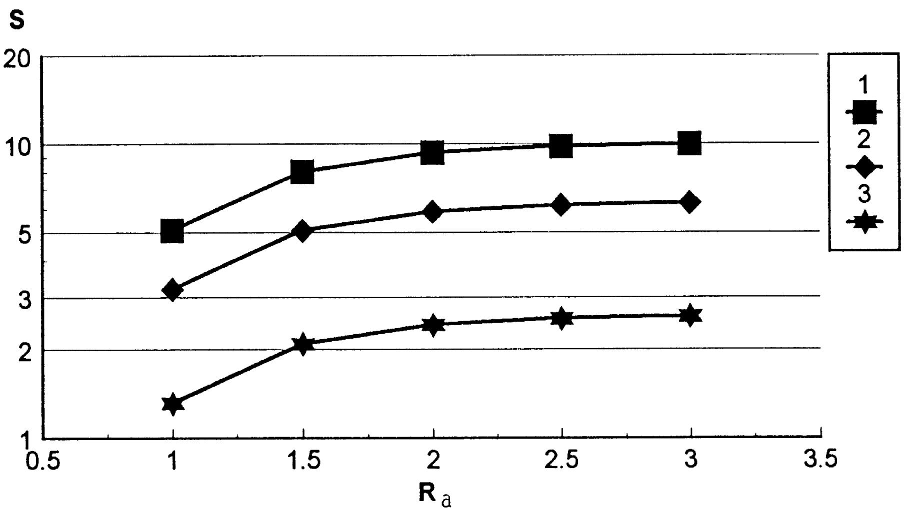

Fig. 2. The inner boundary of the region of pure austenite versus the external tensile stress. Curve 1 : $T = 50°C$, curve 2 : $T = 40°C$, curve 3 : $T = 30°C$.

Consider the region of the plate where the martensitic transformation has started. A closed-form solution can be obtained if the martensitic fraction is excluded from the constitutive eqn (5). The relationship between the rate of the martensitic fraction and that of the effective stress is available from eqn (6) in the form

$$d\xi = kd\sigma \tag{14}$$

where

$$k = -(b_M/d_M)\exp\left[b_M(M_S{}^\circ - T) + (b_M/d_M)\sigma\right] \tag{15}$$

From (9),

$$d\sigma = ad\sigma_r + bd\sigma_\theta \tag{16}$$

where

$$a = (2\sigma_r - \sigma_\theta)/(2\sigma)$$

$$b = (2\sigma_\theta - \sigma_r)/(2\sigma) \tag{17}$$

Then

$$d\xi = Ad\sigma_r + Bd\sigma_\theta \tag{18}$$

where $A = ka$ and $B = kb$.

Equation (18) can be integrated by assuming that A and B remain constant within narrow annular regions of the plate. Physically, this implies a replacement of continuous functions $A = A(r)$ and $B = B(r)$ with piece-wise constant functions. Accordingly, within a region "i" where $r_i < r < r_{i+1}$,

$$\xi = A_i\sigma_r + B_i\sigma_\theta \tag{19}$$

Now eqn (5) can be represented as

$$\begin{aligned} \varepsilon_r &= \{1/[(1-\nu^2)D] + \omega A_i\}\sigma_r + \{-\nu/[(1-\nu^2)D] + \omega B_i\}\sigma_\theta \\ \varepsilon_\theta &= \{-\nu/[(1-\nu^2)D] + \omega A_i\}\sigma_r + \{1/[(1-\nu^2)D] + \omega B_i\}\sigma_\theta \end{aligned} \tag{20}$$

where the modulus of elasticity is a function of state variables. It is customary to assume that

$$D = D_A + \xi(D_M - D_A). \tag{21}$$

Within $r_i < r < r_{i+1}$, the value of D can be assumed constant. Then eqn (20) yields

$$\begin{aligned} \varepsilon_r &= S_{11}\sigma_r + S_{12}\sigma_\theta \\ \varepsilon_\theta &= S_{21}\sigma_r + S_{22}\sigma_\theta \end{aligned} \tag{22}$$

where $S_{mn} = S_{mn}(i)$ are constant compliances corresponding to region "i".

The equation of axisymmetric equilibrium is

$$\sigma_{r,r} + (\sigma_r - \sigma_\theta)/r = 0. \tag{23}$$

The inverse of eqn (22) is:

$$\begin{aligned} \sigma_r &= C_{11}\varepsilon_r + C_{12}\varepsilon_\theta \\ \sigma_\theta &= C_{21}\varepsilon_r + C_{22}\varepsilon_\theta \end{aligned} \tag{24}$$

where the matrix of constant stiffnesses $C_{mn} = C_{mn}(i)$ is obtained as an inverse of the matrix of compliances $S_{mn} = S_{mn}(i)$.

The strain-radial displacement relationships are:

$$\varepsilon_r = u_{,r} \quad \varepsilon_\theta = u/r \tag{25}$$

where u are radial displacements.

Now the equation of equilibrium can be obtained in terms of radial displacements in the form

$$u_{,rr} + F_1 u_{,r}/r - F_2 u/r^2 = 0 \tag{26}$$

where

$$\begin{aligned} F_1 &= 1 + (C_{12} - C_{21})/C_{11} \\ F_2 &= C_{22}/C_{11} \end{aligned} \tag{27}$$

A closed-form solution of eqn (26) is available for each concentric annular region where the martensitic fraction and, accordingly, the effective stress are assumed constant. Then the terms F_1 and F_2 in eqn (26) are constant for the region. The solution of eqn (26) for the i-th region is (Kamke, 1971):

$$u = A_{1i}r^{m(i)} + A_{2i}r^{n(i)} \tag{28}$$

where A_{1i} and A_{2i} are constants of integration and

$$m(i) = (1 - F_{1i} - \mu_i)/2$$

$$n(i) = (1 - F_{1i} + \mu_i)/2$$

$$\mu_i = [(1 - F_{1i})^2 + 4F_{2i}]^{1/2} \tag{29}$$

In eqn (29), F_{1i} and F_{2i} are calculated using eqn (27) with the elements of the matrix of stiffness corresponding to the i-th region. If the value of μ_i is equal to zero,

$$u = r^{t(i)}(A_{1i} + A_{2i} \ln r) \tag{30}$$

where

$$t(i) = (1 - F_{1i})/2 \tag{31}$$

The stresses within the i-th region where radial displacements are given by eqn (28) are:

$$\sigma_r = [C_{11}(i)m(i) + C_{12}(i)]A_{1i}r^{m(i)-1} + [C_{11}(i)n(i) + C_{12}(i)]A_{2i}r^{n(i)-1}$$

$$\sigma_\theta = [C_{21}(i)m(i) + C_{22}(i)]A_{1i}r^{m(i)-1} + [C_{21}(i)n(i) + C_{22}(i)]A_{2i}r^{n(i)-1} \tag{32}$$

where the elements of the matrix of stiffnesses are identified with the i-th region. Expressions for the stresses corresponding to the radial displacement given by eqn (30) are omitted for brevity.

Consider the case where $0 < \xi < 0.99$ along the boundary of the hole. The solution should begin at this boundary ($r = a$) where $\sigma_r = 0$ and a prescribed value of the circumferential stress $\sigma_\theta = \Sigma$ can be identified with the effective stress within the adjacent innermost region. The value of this stress may vary within the range prescribed by inequality (10). Then the martensitic fraction corresponding to the innermost region is calculated by eqn (6). Now the stiffness within the region can be specified. The constants of integration A_{11} and A_{21} ($i = 1$) in eqn (32) are determined from the boundary conditions for the stresses at $r = a$.

At the boundary of the second region ($i = 2$), $r = a + \Delta r$, the stresses σ_r and σ_θ are known from the solution for the first region. These stresses can be used to specify the effective stress within the second region. Then the martensitic fraction and the elements of the matrix of stiffnesses are found. Finally, the constants of integration A_{12} and A_{22} ($i = 2$) are determined from the radial stress and displacement continuity conditions at $r = a + \Delta r$.

The process described above should be continued until the martensitic fraction is equal to zero. This determines the inner boundary of the region of pure austenite. Within this region, the classic elasticity solution is valid. The constants of integration can be specified from the continuity condition for the radial stress and displacement along the inner boundary of the region. Then the value of the external stress can be determined as the value of the radius approaches infinity. This stress represents the load that causes the prescribed value of the circumferential stress at the boundary of the hole.

Note that this solution is based on the assumption that the innermost region contains both martensite and austenite. In the case where the material is completely converted into martensite within the region adjacent to the hole, the stresses in this region can be determined from the solution of the elastic problem with the boundary conditions $\sigma_r = 0$, $\sigma_\theta = \Sigma$ at $r = a$:

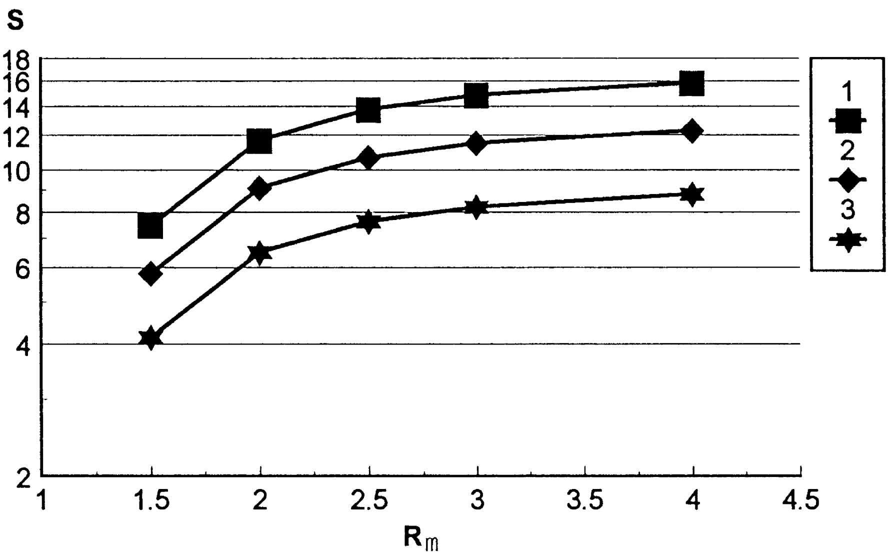

Fig. 3. The outer radius of an annular plate that is completely converted into martensite versus the magnitude of the tensile radial stress. Curve 1 : $T = 50°C$, curve 2 : $T = 40°C$, curve 3 : $T = 30°C$.

$$\sigma_r = \Sigma[1-(a/r)^2]/2$$
$$\sigma_\theta = 2\Sigma[1-(a/r)^2]/2 \tag{33}$$

Obviously, this solution converges to the classical result, if the entire plate material is in the martensitic phase.

The equation for outer boundary of the pure martensite region is determined from eqns (9) (33) and inequality (10) as

$$\frac{\Sigma}{2}[1+3(a/r)^2]^{1/2} = d_M(\ln 0.01/b_M + T - M_S{}^\circ) \tag{34}$$

The results in Fig. 3 represent the external radius of a finite annular plate completely converted to martensite as a result of the radial tensile stress $\sigma_r = s$ applied along the outer boundary. A complete conversion is achieved when $\xi = 0.99$ at the outer boundary $r = r_m$. The nondimensional variable S is defined as in eqn (13) and $R_m = r_m/a$. As is shown in Fig. 3, the outer radius of the plate increases when temperature decreases, as could be expected.

2.3. *Exact solution*

As follows from the solution in the regions of pure martensite and austenite, the ratio of the radial to circumferential stress is given by the same function of the radius, i.e.

$$R = \sigma_r/\sigma_\theta = [1-(a/r)^2]/[1+(a/r)^2] \tag{35}$$

Considering that the regions of pure martensite and austenite encompass the region of mixed martensite and austenite, it is logical to assume that the ratio (35) is not violated within the latter region. This assumption makes an exact solution of the problem possible, as shown below.

Substitution of the circumferential stress from eqn (35) into the equilibrium eqn (23) and a separation of variables yields

$$d\sigma_r/\sigma_r = 2a^2\,dr/(r^3 - a^2 r) \tag{36}$$

The result of integration is:

$$\sigma_r = \exp\{C_1 - \ln[r^2/(r^2 - a^2)]\} \tag{37}$$

where C_1 is a constant of integration.

Now the solution is obtained as follows. The effective stress along the boundary of the region of pure martensite, i.e. $\sigma(r_m)$, can be determined from the right side of inequality (10). Combining eqns (9) and (35) one obtains the corresponding radial stress:

$$\sigma_r(r_m) = \sigma(r_m)[1 - (a/r_m)^2]/[1 + 3(a/r_m)^4]^{1/2} \tag{38}$$

Now the constant of integration C_1 can be specified from eqn (37) at $r = r_m$. For each value of the radius $r > r_m$, the radial stress is given by eqn (37) and the circumferential stress can be determined from eqn (35). The effective stress is subsequently found from eqn (9). When the effective stress is equal to the value given by the left side of inequality (10), the material is in the austenitic phase. The subsequent stress distribution should be based on the elastic solution for the austenitic material.

Note that the solution of the stress problem discussed in this section does not involve a reference to a material constitutive law, except for the nucleation criteria used to specify the boundaries of pure martensite and austenite. This implies that the distribution of radial and circumferential stresses as well as the effective stress are not affected by the temperature-induced martentsitic transformation. However, this transformation affects the material properties and therefore its stiffness and strength. Defining displacements in the plate may be important only if boundary conditions are formulated in terms of displacements (kinematic boundary conditions) what is not the case in the present problem. However, it may be important to know the martensitic fraction in the most vulnerable area of the plate, i.e. in the vicinity of the hole. Accordingly, it is important to consider a situation where the material around the hole experiences a partial transformation. Then the knowledge of the martensitic fraction at $r = a$ becomes critical to estimate the strength.

The problem of the stresses and strength for a plate with a region of partially transformed material adjacent to the hole boundary is solved as follows. Combining a version of eqn (38) where r_m is replaced with r_a and the first eqn (11), one obtains

$$s = \sigma[1 + 3(a/r_a)^4]^{-1/2} \tag{39}$$

This equation can be used to calculate the value of the external stress corresponding to the effective stress that causes transformation at $r = r_a$. Of course, the latter stress is found from (10).

The value of the constant of integration C_1 can be specified from eqn (37) where $r = r_a$ and the stress σ_r corresponding to the effective stress at this location is obtained from eqn (38) where r_m is replaced with r_a. Now the radial stress at $a < r < r_a$ is determined from eqn (37). The corresponding values of the circumferential and effective stresses can be obtained from eqns (35) and (9),

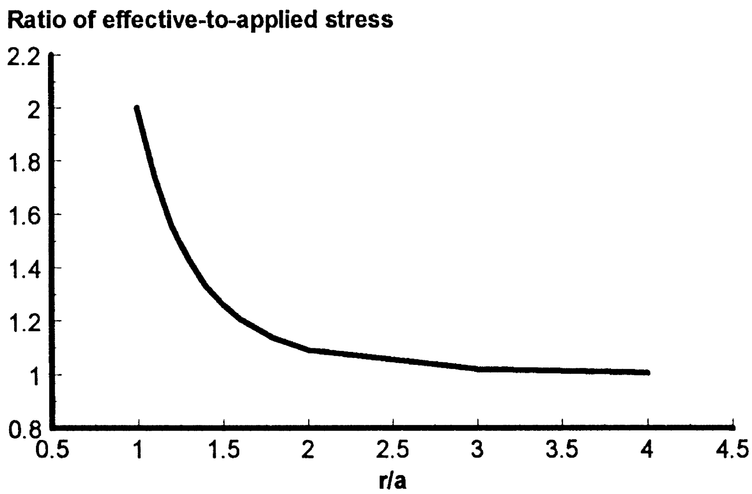

Fig. 4. Distributions of nondimensional effective stresses in a plate at $T = 30°C$.

respectively. Finally, the martensitic fraction is calculated from eqn (6). Note that the circumferential stress obtained from eqn (35) has a singularity at $r = a$. However, the stress concentration factor is still equal to 2, i.e. the exact value of this stress can be predicted.

Results generated using the procedure outlined above are presented in Figs 4 and 5 for a plate with the reference (stress-free) temperature equal to 30°C. At this temperature, the range of

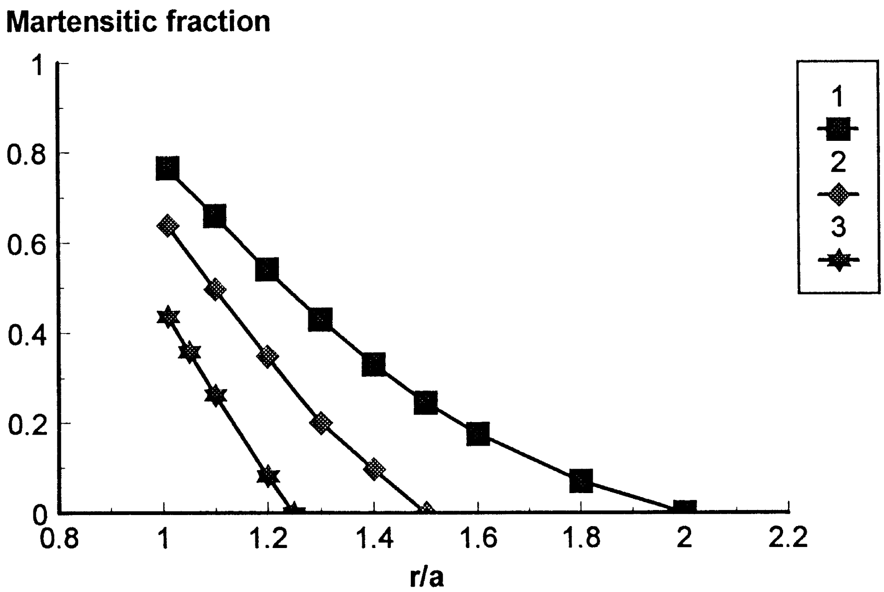

Fig. 5. Distribution of the martensitic fraction in a plate at $T = 30°C$. Case 1 : $s = 72.589$ MPa, case 2 : $s = 62.679$ MPa, case 3 : $s = 52.984$ MPa.

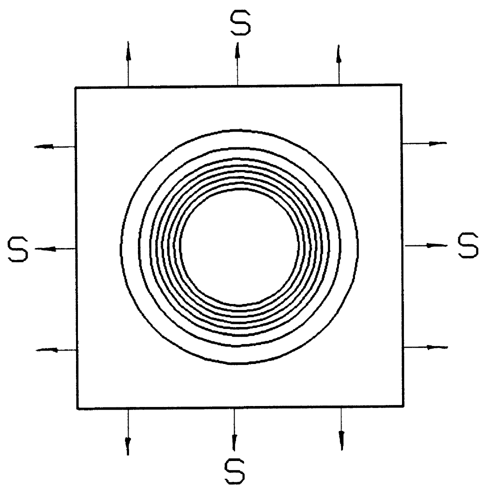

Fig. 6. Contours of constant values of the martensitic fraction at $T = 30°C$ and $s = 72.589$ MPa. The outer concentric circle corresponds to the boundary between the regions of pure and partially transformed austenite. The inner circle (boundary of the hole) corresponds to $\xi = 0.77$. The increments of the martensitic fraction between the circles are equal to 0.11.

stresses corresponding to the martensitic transformation is $79.1 < \sigma < 282.5$ MPa. Three cases are considered corresponding to various external stresses. The nondimensional effective stress–strain curve is unaffected by the magnitude of the external load (of course, the absolute values of stresses increase for larger values of s). However, a larger percentage of material along the boundary of the hole is transformed into martensite as a result of higher external stresses. This implies a lower strength of the material and a higher flexibility of the plate.

It is interesting to consider variations of the martensitic fraction with the radial coordinate. According to Fig. 5, the martensitic fraction seems to be an almost linear function of the coordinate, with the exception corresponding to small values of this fraction. This is also reflected in Fig. 6 where the contours of constant values of the martensitic fraction with an increment equal to 0.11 are at an almost equal distance from each other when the value of the fraction exceeds 0.33. However, at smaller values of the martensitic fraction, the distance between the contours increases reflecting an influence of the adjacent region of pure austenite.

3. Conclusions

The analysis of the stress distribution and the phase transformation in a SMA plate with a circular hole subjected to biaxial tensile stresses is presented in the paper. The solution is based on a two-dimensional version of the Tanaka constitutive theory and the assumption of plane stress. The boundaries of the regions of pure austenite, mixed martensite and austenite and pure martensite are analytically evaluated. It is shown that the region of pure austenite expands toward the hole, if temperature increases. In another example, an annular plate subjected to tensile radial stresses along the outer boundary is considered. The analysis is concerned with the case where the plate material is completely converted into martensite under the load. It is shown that the radius of the plate that is completely converted into martensite increases as temperature decreases.

A closed-form solution is obtained by subdividing the plate into annular regions of constant effective stress and martensitic fraction. An exact solution can be obtained within each region and the constants of integration are subsequently evaluated from the boundary and continuity conditions.

An exact solution of the problem is obtained by assumption that proportional loading is dominant throughout the entire plate. This implies that the ratio of the radial-to-circumferential stress in the region of a partially transformed material is governed by the same law as in the regions of pure martensite and austenite. The solution of the stress problem that is observed in this case is unaffected by the stress-induced martensite. However, the martensitic fraction that is determined from the solution governs the properties of the material around the hole, i.e the strength of the plate. Accordingly, higher external stresses result in a more flexible material around the hole and a lower strength.

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